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Early Universe in the $SU(3)_L \otimes U(1)_X$ Electroweak Models

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Abstract: We present status of the 3-3-1 models and their implications to cosmological evolution such as inflation, phase transitions and sphalerons. The models can deal not only with the issues such as neutrino physics, dark matter, etc, but they are also able to provide quite good agreement with the Standard Cosmology: The inflation happens at the GUT scale, while phase transition has two sequences corresponding two steps of symmetry breaking in the models, namely: SU(3) → SU(2) and SU(2) → U(1). Some bounds on the model parameters are obtained: in the RM331, the mass of the heavy neutral Higgs boson is fixed in the range: 285.56GeV < Mh2 < 1.746TeV and for the doubly charged scalar: 3.32TeV < Mh--< 5.61TeV.

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Introduction

It is well known that our Universe content is 68.3% of Dark Energy (DE), 26.8% of Dark Matter (DM) and of 4.9% of luminous matter (Ade *et al*., 2013). With the unique fact of accelerating Universe, the core origin of Dark Energy is still under question, while the existence of Dark Matter is unambiguous. According to the Standard Cosmology, in the moment at 10^{-36}s after the Big Bang (BB), there was inflation and our Universe has been expanded exponentially. The inflationary scenario solves a number of problems such as the Universe's flatness, horizon, primordial monopole, etc. It is well known that there is no anti-matter in our Universe, or other word speaking: at present there exists a Baryon Asymmetry of Universe (BAU). The baryon number vanishes ($n_B = 0$) at the BB and this conflicts with the present BAU. Nowadays, the BAU is one of the greatest challenges in Physics and any physical model has to give an explanation. The BAU is realized if three Sakharov's conditions are satisfied (Sakharov, 1967; Mukhanov, 2005):

- B violation,
- C and CP violations,
- deviation from thermal equilibrium

Over the half of Century, the Standard Model (SM) of the electromagnetic, weak and strong interactions successfully possesses a great experimental examinations and stands for future development. Despite its great success, the model still contains a number of unresolved problems such as the generation number of quarks and leptons, the neutrino mass and mixing, the electric charge quantization, the existence of about one quarter of DM, etc. The aforementioned problems require that the SM must be extended.

Among the extensions beyond the SM, the models based on $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge group (Pisano and Pleitez, 1993; Singer *et al*., 1980) have some interesting features including the ability to explain the generation problem (Pisano and Pleitez, 1993; Singer *et al*., 1980) and the electric charge quantization (Pires and Ravinez, 1998). It is noted that in this scheme the gauge couplings can be unified at the scale of order TeV without super symmetry (Boucenna *et al*., 2015). The 3-3-1 models have two interesting properties needed for the mentioned aim, namely: first, the lepton-number violation due to the fact that lepton and anti-lepton are put in the triplet (Chang and Long, 2006). Second, one generation of quarks transforms differently from other two. This leads to the flavor changing neutral current at the tree level mediated by new Z′ gauge boson (Long and Van, 1999).

The 3-3-1 models have been considered in aspects of collider physics (Cieza Montalvo *et al*., 2013; 2012; Yue *et al*., 2013; Caetano *et al*., 2013), muon anomalous magnetic moments (Ky *et al*., 2000), neutrino physics (Dong *et al*., 2011), DM (Fregolente and Tonasse, 2003; Dong *et al*., 2014).... In this review I will concentrate on Early Universe aspects of the models.

This study is organized as follows. In Section II we give a brief review of the 3-3-1 models and their modified versions. In Section III, the cosmological inflation in the super symmetric economical 3-3-1 model is presented. In Section IV, we investigate the structure of the Electroweak Phase Transition (EWPT) sequence in the 3-3-1 models with minimal Higgs sector, namely the reduced minimal 3-3-1 model (RM331) and the economical 3-3-1 model (E331), find the parameter ranges where the EWPTs are the strongly first-order to provide B violation necessary for baryogenesis and show the constraints on the mass of the charged Higgs boson. Section V is devoted for sphalerons in the reduced minimal 3-3-1 model. Finally, in Sec. VI we give conclusion on the possibility to describe cosmological evolution in the framework of the 3-3-1 models.

The Models

In the mentioned models, the strong interaction keeps the same as in the SM, while the electroweak part associated with $SU(3)_L \otimes U(1)_X$ has two diagonal generators T_3 and T_8 from which the electric charge operator is based on:

$$Q = T_3 + \beta T_8 + X \quad (1)$$

The coefficient (=1) at the T_3 is defined to make the 3-3-1 models embed the SM. The lepton arrangement will define the parameter β which distinguishes two main versions: The minimal version with $\beta = \sqrt{3}$ and the version with neutral leptons/neutrinos $\beta = -1\sqrt{3}$ at the bottom of the triplet.

The Minimal 3-3-1 Model

The minimal version (Pisano and Pleitez, 1993) contains lepton triplet in the form:

$$f_L = (\nu_l, l, l^c)_L^T \sim (1,3,0) \quad (2)$$

Two first quark generations are in anti-triplet and the third one is in titriplet:

$$\begin{aligned} &Q_{iL} = (d_{iL} - u_{iL}, D_{iL})^T \sim \left(3, \bar{3}, -\frac{1}{3}\right) \\ &u_{iR} \sim (3,1,2,3), d_{iR} \sim (3,1-1/3), D_{iR} \sim (3,1,-4/3), i = 1,2, \\ &Q_{3L} = (u_{3L}, d_{3L}, T_L)^T \sim (3,3,2/3) \\ &u_{3R} \sim (3,1,2/3), d_{3R} \sim (3,1,-1/3), T_R \sim (3,1,5/3) \end{aligned} \quad (3)$$

To provide masses for all quarks and lepton, the Higgs sector needs three scalar triplets and one sextet:

$$\begin{aligned} &X = \left(X_1^-, X_2^{--}, X_3^0\right)^T \sim (1,3,-1) \\ &\eta = \left(\eta_1^0, \eta_2^-, \eta_3^+\right)^T \sim (1,3,0) \\ &\rho = \left(\rho_1^+, \rho_2^0, \rho_3^{++}\right)^T \sim (1,3,1) \\ &S \sim (1,6,0) \end{aligned} \quad (4)$$

with

$$\begin{aligned} VEV &: \langle \rho_2^0 \rangle = \upsilon / \sqrt{2}, \langle \eta_1^0 \rangle \\ &= u / \sqrt{2}, \langle \eta_3^0 \rangle = \omega / \sqrt{2} \, and \, \langle S_{23}^0 \rangle \dot{\upsilon} / \sqrt{2} \end{aligned}$$

The gauge sector of this model contains five new gauge bosons: One neutral Z′ and two bileptons carrying lepton number 2: $Y^{\pm}$ and $X^{\pm\pm}$. In (2), lepton and antilepton lie in the same triplet and this leads to lepton number violations in the model. Hence, it is better to deal with a new conserved charge L commuting with the gauge symmetry (Chang and Long, 2006):

$$L = \frac{4}{\sqrt{3}} T_8 + L \quad (5)$$

The exotic quarks T and D_i have the electric charges, respectively, 5/3 and -4/3 and carry both baryon and lepton numbers L = ±2.

The singly charged bilepton is responsible for the wrong muon decay:

$$\mu \to e + \nu_e + \tilde{\nu}_\mu$$

While the doubly charged bilepton with decay:

$$X^{--} \to ll$$

Provides four leptons at the final states which is characteristic feature of the model. The model provides an interesting prediction for the Weinberg angle:

$$sin^2\theta_w(M_{Z'}) \leq \frac{1}{4}$$

Besides the complication in the Higgs sector, the model also has one problem that it losses perturbative property at the scale above 5 TeV (Dias *et al*., 2005).

The above Higgs sector is complicated; and recently it is reduced to the minimal with only two Higgs triplets (Dong *et al*., 2014; Ferreira *et al*., 2011). If the triplet ρ and χ are used then the model is called reduced minimal 3-3-1 model (Ferreira *et al*., 2011), while ρ is replaced by η then it is called simple 3-3-1 model (S331) (Dong *et al*., 2014).

It has been recently shown that due to the ρ parameter and the Landau pole, the minimal and its reduced version should be ruled out (Dong and Si, 2014). It is noted that the RM331 has nonrenormalizable effective interactions, so situation has to be considered carefully.

The 3-3-1 Model with Right-Handed Neutrinos

Leptons are in triplet (Singer *et al*., 1980):

$$f_L^a = (\nu_L^a, e_L^a, (N_L)^a)^T \sim (1,3,-1/3), e_R^a \sim (1,1,-1) \quad (6)$$

where, a = 1, 2, 3 is a generation index and N_L can be right-handed neutrino or neutral lepton. Two first generations of quarks are in antitriplets and the third one is in triplet:

$$\begin{aligned} &Q_{iL} = (d_{iL}, -u_{iL}, -D_{iL})^T \sim (3,\bar{3},0), \\ &u_{iR} \sim (3,1,2/3), d_{iR} \sim (3,1,-1/3), D_{iR} \sim (3,1,-1/3), i = 1,2, \\ &Q_{3L} = (u_{3L}, d_{3L}, T_L)^T \sim (3,3,1/3), \\ &u_{3R} \sim (3,1,2/3), d_{3R} \sim (3,1,-1/3) T_R \sim (3,1,2/3) \end{aligned} \tag{7}$$

The model with neutral lepton/neutrino $\beta = -1\sqrt{3}$ needs three scalar triplets to provide all fermions masses and the same for Spontaneous Symmetry Breaking (SSB):

$$\begin{aligned} &X = (X^0, X^-, X^{'0})^T \sim (1,3,-1) \\ &\rho = (\rho^+, \rho^0, \rho^{'+})^T \sim (1,3,2) \\ &\eta = (\eta^0, \eta^-, \eta^{'0}) \sim (1,3,-1) \end{aligned} \tag{8}$$

The exotic quarks T and D_i have electric charges as usual one, i.e., 2/3 and -1/3, respectively and carry both baryon and lepton numbers L = ±2. The new gauge bosons are: the neutral Z′ and two bileptons carrying lepton number 2: $Y^{\pm}$ and X^0. The neutral bilepton X^0 is non-Hermitian and is responsible for neutrino oscillation (Long and Inami, 2000).

Note that two Higgs triplets η and χ have the same structure, so ones can reduce number of Higgs triplets from three to two, namely we can use only ρ and χ to produce masses for quarks and leptons; and resulting model is called economical 3-3-1 model (Ponce *et al*., 2003). As in the RM331, the nonrenormalizable interactions, in this case, are needed for production of quark masses (Ponce *et al*., 2003).

Cosmological Inflation in the Super Symmetric Economical 3-3-1 Models

The discovery of the 2.7K microwave background radiation arriving from the farthest reaches of the Universe, gained widespread acceptance, is positive point of the hot-universe theory, where the inflationary scenario (Guth, 1981; Linde, 1983) plays very important role. Cosmological Inflation (CI) can give solutions for above mentioned problems, hence it is a possible theory of the origin of all structures in the Universe, including ourselves!

With above reasons, any beyond standard model has to have the cosmological inflation happened at the interval of 10^{-36}-10^{-34}s after the BB. With that moment, the energy scale of CI is about 10^{15} GeV. In (Huong and Long, 2010), the CI was considered in the framework of the super symmetric economical 3-3-1 model (SE331) and a reason is the following: The E331 is very simple, but there is no candidate for in flat on-a key element of CI. The SE331 has some advantages such as there are more scalar fields which can play a role of the inflaton and the Higgs sector is very constrained.

A supersymmetric version of the minimal 3-3-1 model has been constructed in (Montero *et al*., 2004) and its scalar sector was studied in (Duong and Ma, 1993). Lepton masses in the framework of the above-mentioned model were presented in (Montero *et al*., 2002), while potential discovery of supersymmetric particles was studied in (Capdequi-Peyranere and Rodriguez, 2002). In (Long and Pal, 1998), the R-parity violating interaction was applied for instability of the proton. A supersymmetric RM331 was presented in (Huong *et al*., 2013).

The supersymmetric version of the 3-3-1 model with right-handed neutrinos has already been constructed in (Montero *et al*., 2004). The scalar sector was considered in (Huong *et al*., 2005) and neutrino mass was studied in (Dong *et al*., 2006). A supersymmetric version of the economical 3-3-1 model has been constructed in (Dong *et al*., 2007). Some interesting features such as Higgs bosons with masses equal to that of the gauge bosons: $W(m^2_{e_1^+} = m^2_W)$ and the bileptons X and Y $(m^2_{\xi_4^\pm} = m^2_Y)$, have been pointed out in (Dong *et al*., 2008). Sfermions in this model have been considered in (Dong *et al*., 2007). In (Huong and Long, 2008) it was shown that bino-like neutralino can be a candidate for DM.

In (Huong and Long, 2010), the authors have constructed a hybrid inflationary scheme based on a realistic supersymmetric $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model by adding a singlet superfield ϕ which plays the role of the inflaton, namely the inflaton superfield.

We remind that the existence of a $U(1)_Z$ does not belong to the MSSM and it spontaneously breaks down at the scale M_X by Higgs superfield ϕ, which is singlet under the MSSM. The inflaton superfield couples with this pair of Higgs superfields. Therefore, the additional global supersymmetric renorrmalizable superpotential for the inflation sector is chosen to be (Copeland *et al*., 1994; Dvali *et al*., 1994):

$$W_{inf}(\Phi, X, X') = a\Phi XX' - \mu^2\Phi \tag{9}$$

The super potential given by (9) is the most general potential consistent with a continuous R symmetry under which $\phi \to \phi e^{i\gamma}\phi$, $W \to \phi e^{i\gamma} W$, while the product $\chi\chi'$ is invariant (Dvali *et al*., 1994; Linde and Riotto, 1997).

By a suitable redefinition of complex fields μ^2, α are chosen to be positive real constants and the ratio $\frac{\mu}{\sqrt{a}}$ sets the $U(1)_Z$ symmetry breaking scale MX. The most general super potential consistent with a continuous R-symmetry is given by:

$$W_{tot} = W_R + W_{inf}(\Phi, X, X') \tag{10}$$

With the super potential given in (9), the Higgs scalar potential takes the form:

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$$V_{tot} = \sum_i \left|F_i\right|^2 + \frac{1}{2}\sum_a \left|D_a\right|^2 + V_{soft}$$

where, i runs from 1 to the total number of the chiral super fields in W_{tot}, while V_{soft} contains all the soft terms generated by super symmetry breaking at the low energy. Hence, the Higgs potential becomes:

$$V_{tot} = \left|\mu_X + a\Phi\right|^2 \left|X'\right|^2 + \left|\mu_X + a\Phi\right|^2 \left|X'\right|^2 + \left|aXX' - \mu^2\right|^2 + \\ + \left|\mu_\rho \rho\right|^2 + \left|\mu_{\rho'}\rho'\right|^2 + \frac{1}{2}\sum_a \left|D_a\right|^2 + V_{soft}$$

The first derivatives $\frac{\partial V_{tot}}{\partial_\rho}, \frac{\partial V_{tot}}{\partial \rho'}$ are independent of χ, χ′, ϕ and the fields ρ, ρ′ will stay in their minimum independently of what the fields χ, χ′, ϕ do. If we are mainly interested in what is happening above the electroweak scale and hence we do not take into account the dimensional Higgs multiplets ρ, ρ′. Then, the Higgs scalar potential is given by:

$$V_{inf} = \left|\mu_X + a\Phi\right|^2 \left|X'\right|^2 + \left|\mu_X + a\Phi\right|^2 \left|X\right|^2 + \left|aXX' - \mu^2\right|^2 + \\ \frac{1}{2}\left(g\sum_a X^* T^a X\right)^2 + \frac{1}{2}\left(g\sum_a X'^* T^a X\right)^2 \tag{11}$$

Let us denote:

$$\mu_X + a\Phi \equiv \phi S \tag{12}$$

where, β is some constant and S is a new field, the Higgs potential (11) can be rewritten as:

$$V_{inf} = \beta^2 \left|S\right|^2 \left(\left|X\right|^2 + \left|X'\right|^2\right) + \left|aXX' - \mu^2\right|^2 \\ \frac{1}{2}\left(g\sum_a X^* T^a X\right)^2 + \frac{1}{2}\left(g\sum_a X'^* T^a X'\right)^2$$

When D term vanishes along its direction, the potential contains only F term and has the form:

$$V_{inf} = \beta^2 \left|S\right|^2 \left(\left|X\right|^2 + \left|X'\right|^2\right) + \left|aXX' - \mu^2\right|^2 \tag{13}$$

From (13), it is clear that V_{inf} has an unique super symmetric minimum corresponding to:

$$\langle S \rangle = 0 \\ M_X \equiv \langle X \rangle = \langle X' \rangle = \frac{\mu}{\sqrt{2}} \tag{14}$$

The ratio $\frac{\mu}{\sqrt{a}}$ sets the $U(1)_Z$ symmetry breaking M_X, but Equtaion (14) is global minimum and supersymmetry is not violated (Dvali *et al.*,1994). Hence, inflation can take place but supersymmetry is not broken. This is F term inflation (Jeannerot, 1997).

We assume that the initial value for the inflaton field is much greater than its critical value S_c. For $|S| > |S_c| \equiv \frac{\mu}{\sqrt{a}}$ the potential is very flat in the $|S|$ direction and the χ, χ′ fields settle down to the local minimum of the potential, χ = χ′ = 0, but it does not drive S to its minimum value. The universe is dominated by a nonzero vacuum energy density, $V_0^{\frac{1}{4}} = \mu$, which can lead to an exponential expanding, inflation starts and supersymmetry is broken.

By the Coleman-Weinberg formula in (Coleman and Weinberg, 1973), at the one-loop level, the effective potential along the inflaton direction is given by:

$$\Delta V = \frac{1}{64\pi^2}\sum_i (-1)^F m_i^4 In\left(\frac{m_i^2}{\Lambda^2}\right)$$

where, F = -1 for the fermionic fields and F = 1 for the bosonic fields. The coefficient $(-1)^F$ shows that bosons and fermions give opposite contributions. The sum runs over each degree of freedom i with mass mi and _ is a renormalization scale.

The effective potential (along the inflationary trajectory $S > S_c$, χ = χ′ = 0) is given by:

$$V_{eff}(S) = \mu^4 + \frac{3}{16\pi^2} \\ \left[2\beta^4 \frac{\mu^4}{a^2} In \frac{\beta^2 |S|^2}{\Lambda^2} + (\beta^2 |S|^2 + a\mu^2)^2 In\left(1 + \frac{a\mu^2}{\beta^2 |S|^2}\right) + \\ +(\beta^2 |S|^2 - a\mu^2)^2 In\left(1 - \frac{a\mu^2}{\beta^2 |S|^2}\right) \right] \tag{15}$$

It is to be noted that for $S > S_c$, the universe is dominated by the false vacuum energy μ^4. When S field drops to S_c, then the GUT phase transition happens. At the end of inflation, the inflaton field does not need to coincide with the GUT phase transition. The end of inflation can be supposed to be on a region of the potential which satisfies the flatness conditions (see, for example, (Lyth and Riotto, 1999)):

$$\in \ll 1, \eta \ll 1 \tag{16}$$

where, we have used the conventional notations:

$$\in \equiv \frac{M_P^2}{16\pi}\left(\frac{V'}{V}\right), \eta \frac{M_P^2}{8\pi}\frac{V''}{V} \tag{17}$$

where, primes denote a derivative with respective to S.

To compare with observational COBE data, we use the slow-roll approximation with parameters: $\in$ and η. The first condition in (16): $\in \ll 1$ indicates that the density ρ is close to V and is slowly varying. As a result, the Hubble parameter H is slowly varying, which implies that one can write $a \propto e^{Ht}$ at least over a Hubble time or so. The second condition $\eta \ll 1$ is a result of the first condition plus the slow-roll approximation. The conditional phase may end before the GUT transition if the flatness conditions (16) are violated at some point $S>S_c$.

Let us denote a dimensionless variable:

$$y \equiv \frac{\beta|S|}{aS_c} \tag{18}$$

Imposing the condition $\alpha = \beta$, which means that $|\phi| \approx |S| \gg \mu_\chi$, we get then:

$$\begin{aligned} \in &= \left(\frac{3a^2M_P}{4\pi^2M_X}\right)^2 \frac{1}{16\pi}\left[\begin{array}{l} y(y^2-1)In\left(1-\frac{1}{y^2}\right)+ \\ +y(y^2+1)In\left(1+\frac{1}{y^2}\right)\end{array}\right]^2, \\ \eta &= \left(\frac{aM_P}{4\pi M_X}\right)^2 \frac{3}{2\pi}\left[\begin{array}{l} (3y^2+1)In\left(1+\frac{1}{y^2}\right) \\ +(3y^2-1)In\left(1-\frac{1}{y^2}\right)\end{array}\right] \end{aligned} \tag{19}$$

The chaotic inflation driven by the ϕ^3 is in good agreement with the WMAP data, while for the ϕ^4 potential, the situation is negative.

The above model cannot resolve the horizon/flatness problems of the BB cosmology and violates the slow-roll conditions $\eta \ll 1$ (the η problem). To deal with these problems, we should consider the F-term inflation with minimal Kahler potential.

The F-term inflation with Kahler potential is defined by:

$$W_{stand}(\Phi, X, X') = a\hat{S}(\hat{X}\hat{X}' - M_X^2) \tag{20}$$

Keeping in mind that $K = \sum_a |\phi_a|^2$, we obtain the scalar potential:

$$\begin{aligned} V_F^m &= 2a^2S^2\phi^2\left[1+\frac{S^2+2\phi^2}{m_P^2}+\frac{(S^2+2\phi^2)^2}{2m_P^4}\right] \\ &+a^2(\phi^2-M_X^2)^2\left(1+2\frac{\phi^2}{m_P^2}+\frac{S^4}{2m_P^4}+2\frac{\phi^4}{m_P^4}\right) \end{aligned} \tag{21}$$

where, we have assumed that $|\varphi|^2 = |\varphi'|^2$.

Let us consider how does this factor change the result. As we know, the slow-roll parameter is defined as:

$$\eta = m_p^2\left(\frac{V''}{V}\right)$$

where, the prime refers to derivative with respect to S. The super gravity scalar potential for $S>S_c$ is given by:

$$V_o = a^2M_X^4 + \frac{a^2M_X^4}{2m_p^4}S^4 \tag{22}$$

From (22), it follows derivative of $V: V'' \simeq \frac{a^2M_X^4}{2m_p^4}S^2 and \eta = \frac{1}{2m_p^2}S^2 \ll 1$. Therefore, the η-problem is overcome.

The potential given in (22) does not contain a term which can drive S to its minimum value, so we have to consider the effective potential. In this case, the spectral index n is given by:

$$\begin{aligned} n &= 1-6\in+2\eta = 1-\frac{3a^2}{512\pi^3}[x^2(16+9a)^2-54a^2\xi+ \\ &6x^8(-40+9a^2)\zeta+16x^4(-5+9a^2)\zeta] \end{aligned} \tag{23}$$

where, $\zeta \equiv \frac{M_X^2}{M_P^2}$.

Taking into account the WMAP data, we conclude that the value of e-folding number N_Q must be larger than 45 and get bounds on the values of coupling α and ζ, which are presented in Table 1.

It is interesting to note that due the inflaton with mass in the GUT scale, the model can provide masses for neutrino different from ones without inflationary scenario. With the help of the lepton-number-violating interactions among the inflaton and right-handed neutrinos, the non-thermal leptogenesis scenario is followed (Huong and Long, 2011).

In recent work (Huong *et al*., 2015), the authors have considered the inflationary scenario and leptogenesis in newly proposed 3-3-1-1 model. Here, the scalar field that spontaneously breaks the $U(1)_N$ symmetry plays a role of inflaton.

To finish this section, we emphasize that the 3-3-1 models can provide the inflationary scenario or cosmological evolution of our Universe.

Table 1. Bounds on the parameter ζ and coupling

α	10^{-3}	10^{-4}	10^{-5}	10^{-6}
ζ	25×10^{-6}	25×10^{-7}	25×10^{-9}	3×10^{-11}

Electroweak Phase Transition in 3-3-1 Models

It is known that if baryon number is conserved and is equal to zero, it will equal to zero forever. If baryon number does not satisfy any conservation law, it vanishes in the state of thermal equilibrium. Therefore we need the third Sakharov's condition. The second condition is appropriate for ensuring a different decay rate for particles and antiparticles (Mukhanov, 2005). The electroweak phase transition is the transition between symmetric phase to asymmetric phase in order to generate mass for particles. Hence, the phase transition is related to the mass of the Higgs boson (Mukhanov, 2005).

In the basic model of particles, the first and second conditions can be satisfied, but conditions on thermal imbalance is difficult to satisfy. So the analysis of the third condition is the only approach at present in order to explain the baryon asymmetry.

Why is the first order phase transition? For very large temperature, the effective potential has only one minimum at the zero. As temperature drops below the critical temperature (T_c), the second minimum appears. If the two minimums are separated by a potential barrier, the phase transition occurs with bubble nucleation. Inside the bubbles, the scalar field acquires a nonzero expectation value. If the bubble nucleation rate exceeds the universe's expansion rate, the bubbles collide and eventually fill all space. Such a transition is called the first order phase transition. It is very violent and one can expect large deviations from thermal equilibrium (Mukhanov, 2005). The other possible scenario takes place if the two minimums are never separated by a potential barrier. The phase transition is a smooth transition or the second order phase transition.

Phase Transition in Reduced Minimal 3-3-1 Model

For the SM, although the EWPT strength is larger than unity at the electroweak scale, it is still too weak for the mass of the Higgs boson to be compatible with current experimental limits (Mukhanov, 2005; Kajantie *et al*., 1996); this suggests that Electroweak Baryogenesis (EWBG) requires new physics beyond the SM at the weak scale (Bastero-Gil *et al*., 2000). Many extensions such as the two-Higgs-doublet model or Minimal Super symmetric Standard Model have a more strongly first-order phase transition and the new sources of CP violation, which are necessary to account for the BAU; triggers for the first-order phase transition in these models are heavy bosons or DM candidates (Cline *et al*., 2009; Kanemura *et al*., 2005; Ham *et al*., 2010; Das *et al*., 2010).

To start, let us consider the hight-temperature effective potential:

$$V_{eff} = D.(T^2 - T_0'^2)\upsilon^2 - E.T\upsilon^3 + \frac{\lambda_T}{4}\upsilon^4$$

where, v is the VEV of Higgs. In order to have the strongly first-order phase transition, the strength of phase transition has to be larger than 1, i.e., $\frac{\upsilon_c}{T_c} \geq 1$.

The phase transition has been firstly investigated in the SM. But the difficulty of the SM is that the strength of the first-order electroweak phase transition, which must be larger than 1 at the electroweak scale, appears too weak for the experimentally allowed mass of the SM scalar Higgs boson (Mukhanov, 2005; Kajantie *et al*., 1996). Therefore, it seems that EWBG requires a new physics beyond the SM at weak scale (Bastero-Gil *et al*., 2000).

With the discovery of the Higgs boson, the study of phase transitions in the particle models is simplified: only to determine the order of phase transition. This opens a lot of hope for the extended models in examining the electroweak phase transition. The 3-3-1 models must have at least two Higgs triplets (Ferreira *et al*., 2011; Ponce *et al*., 2003). Therefore, the number of bosons in the 3-3-1 models will many more than in the SM and symmetry breaking structure is different to the SM.

The physical scalar spectrum of the RM331 model is composed by a doubly charged scalar h^{++} and two neutral scalars h_1 and h_2 (Ferreira *et al*., 2011). These new particles and exotic quarks can be triggers for the first order phase transition.

From the Higgs potential we can obtain V0 that depends on VEVs as the following:

$$V_0(\upsilon_X, \upsilon_\rho) = \mu_1^2\upsilon_X^2 + \mu_2^2\upsilon_\rho^2 + \lambda_1\upsilon_X^4 + \lambda_2\upsilon_\rho^4 + (\lambda_3 + \lambda_4)\upsilon_X^2\upsilon_\rho^2$$

The effective potential being a function of VEVs and temperature has the form:

$$V = V_0(\upsilon_X, \upsilon_\rho) + \sum M_{boson}^2(\upsilon_X, \upsilon_\rho) W^\mu W_\mu + \sum m_{fermion}(\upsilon_X, \upsilon_\rho)\overline{f_L^c} f_L^c$$

Averaging over space, we obtain:

$$V = V_0(\upsilon_X, \upsilon_\rho) + \sum M_{boson}^2(\upsilon_X, \upsilon_\rho) W^\mu W_\mu + \sum m_{fermion}(\upsilon_X, \upsilon_\rho)\left\langle \overline{f_L^c} f_L^c \right\rangle$$

where, W^μ runs over all gauge fields. The RM331 has the following gauge bosons: Two like the SM bosons Z_1, $W^\pm$ and the new heavy neutral boson Z_2, the singly and doubly charged boson $U^{\pm\pm}$ and $V^\pm$. Two doubly charged Higgs h^{++} and h^{--}, one heavy neutral Higgs h_2 and one like-SM Higgs h_1. Using Bose-Einstein and Fermi-Dirac distributions for bosons and fermions, we can obtain the effective potential in the RM331 as follows:

$$V_{eff}^{RM\,331} = V_0\left(\upsilon_X, \upsilon_\rho\right) + \frac{3}{64\lambda^2}\left(m_{Z_1}^4 In\frac{m_{Z_1}^2}{Q^2} + m_{Z_2} In\frac{m_{Z_2}^2}{Q^2} + 2m_w^4 In\frac{m_w^2}{Q^2} - 4m_t^4 In\frac{m_t^2}{Q^2} - 12m_Q^4 In\frac{m_Q^2}{Q^2}\right)$$
$$+\frac{1}{64\pi^2}\left(m_{h_2}^4 In\frac{m_{h_2}^2}{Q^2} + 2m_{h++}^4 In\frac{m_{h++}^2}{Q^2}\right) + \frac{3}{64\pi^2}\left(2m_U^4 In\frac{m_U^2}{Q^2} + 2m_V^4 In\frac{m_V^2}{Q^2}\right)$$
$$+\frac{T^4}{4\pi^2}\left[F - \left(\pi\frac{m_{h_2}}{T}\right) + 2F_-\left(\frac{m_{h++}}{T}\right)\right] + \frac{3T^4}{4\pi^2}\left[4F - \left(\pi\frac{m_t}{T}\right) + 12F_+\left(\frac{m_Q}{T}\right)\right]$$
$$+\frac{3T^4}{4\pi^2}\left[F - \left(\frac{m_{Z_1}}{T}\right) + F_-\left(\frac{m_{Z_2}}{T}\right) + 2F_-\left(\frac{m_W}{T}\right) + 2F_-\left(\frac{m_U}{T}\right) + 2F_-\left(\frac{m_V}{T}\right)\right]$$

where:

$$F_{\mp}\left(\frac{m_\phi}{T}\right) = \int_0^{\frac{m_\phi}{T}} aJ_{\mp}^{(1)}(a,0)da$$

$$J_{\mp}^{(1)}(a,0) = 2\int_a^\infty \frac{(x^2-a^2)}{e^x \mp 1}dx$$

The effective potential can be rewritten as follows:

$$V_{eff} = V_0 + V_{eff}^{hard} + V_{eff}^{light}$$

where:

$$V_{eff}^{hard} = \frac{3}{64\lambda^2}\left(m_{Z_2}^4 In\frac{m_{Z_2}^2}{Q^2} + m_{h_2}^4 In\frac{m_{h_2}^2}{Q^2} + 2m_{h++}^4 In\frac{m_{h++}^2}{Q^2}\right)$$
$$+\frac{3}{64\pi^2}\left(2m_U^4 In\frac{m_U^2}{Q^2} + 2m_V^4 In\frac{m_V^2}{Q^2} - 12m_Q^4 In\frac{m_Q^2}{Q^2}\right)$$
$$+\frac{T^4}{4\pi^2}\left(F - \left(\frac{m_{h2}}{T}\right) + 2F_-\left(\frac{m_{h++}}{T}\right)\right)$$
$$+\frac{3T^4}{4\pi^2}\left(F - \left(\frac{m_{Z2}}{T}\right) + 2F_-\left(\frac{m_U}{T}\right) + 2F - \left(\frac{m_V}{T}\right) + 12F_+\left(\frac{m_Q}{T}\right)\right)$$

And:

$$V_{eff}^{light} = \frac{3}{64\lambda^2}\left(m_{Z_1}^4 In\frac{m_{Z_1}^2}{Q^2} + 2m_w^4 In\frac{m_w^2}{Q^2} + 2m_t^4 In\frac{m_t^2}{Q^2}\right)$$
$$+\frac{3T^4}{4\pi^2}\left(F_-\left(\frac{m_{z1}}{T}\right) + 2F_-\left(\frac{m_w}{T}\right) + 4F_+\left(\frac{m_t}{T}\right)\right)$$

Here V_{eff}^{light} is like the effective potential of the SM, while V_{eff}^{hard} is contributions from heavy particles. We expect that V_{eff}^{hard} contributes heavily in the EWPT.

The symmetry breaking in the RM331 can take place sequentially. Because two scales of symmetry breaking are very different, $\upsilon_{\chi 0} \gg \upsilon_{\rho 0}$ ($\upsilon_{\chi 0} \sim 4-5$ TeV, $\upsilon_{\rho 0} = 246$ GeV) and because of the accelerating universe, the symmetry breaking SU(3) $\rightarrow$ SU(2) takes place before the symmetry breaking SU(2) $\rightarrow$ U(1). The symmetry breaking SU(3) $\rightarrow$ SU(2) through χ_0, generates the masses of the heavy gauge bosons such as $U^{\pm\pm}$, $V^{\pm}$, Z_2 and exotic quarks.

Through the boson mass formulations in the above sections, we see that boson $V^{\pm}$ only involves in the phase transition SU(3) $\rightarrow$ SU(2). Z_1, $W^{\pm}$ and h1 only involve in the phase transition SU(2) $\rightarrow$U(1). However, $U^{\pm\pm}$, Z_2 and h^{--} involve in both two phase transitions. The first one is the phase transition SU(3) $\rightarrow$ SU(2). This phase transition involves exotic quarks, heavy bosons, without involvement of the SM particles, so υ_ρ is omitted in this phase transition. The effective potential can be rewritten as follows (Phong *et al*., 2013):

$$V_{SU(3)\rightarrow SU(2)}^{eff} = D'(T^2 - T_0'^2)\upsilon_X^2 - E'T\upsilon_X^3 + \frac{\lambda_T'}{4}\upsilon_X^4$$

The minimum conditions are:

$$V_{eff}(X_0) = 0; V_{eff}'(X_0) =; V_{eff}''(X_0) = m_{h_2}^2$$

where:

$$D' = \frac{1}{24\upsilon_{X0}{}^2}\{6m_U^2 + 3m_{Z_2}^2 + 6m_V^2 + 18m_Q^2 + 2m_{h\pm}^2\},$$
$$T_0'^2 = \frac{1}{D}\left\{\frac{1}{4}m_{h_2}^2 - \frac{1}{32\pi^2\upsilon_{X0}^2}(6m_U^4 + 3m_{Z_2}^4 + 6m_V^4 - 36m_Q^4 + 2m_{h\pm}^4)\right\},$$
$$E' = \frac{1}{12\pi\upsilon_{X0}^3}(6m_U^3 + 3m_{Z_2}^3 + 6m_V^3 + 2m_{h\pm}^3),$$
$$\lambda_T' = \frac{m_{h2}^2}{2\upsilon_{X0}^2}\left\{1 - \frac{1}{8\pi^2\upsilon_{X0}^2 m_{h2}^2}\left[\begin{matrix}6m_V^4 In\frac{m_V^2}{bT^2} + 3m_{Z_2}^4 In\frac{m_{Z_2}^2}{bT^2} + 6m_U^4 \\ In\frac{m_U^2}{bT^2} - 36m_Q^4 In\frac{m_Q^2}{b_F T^2} + 2m_{h\pm}^4 In\frac{m_{h\pm}^2}{bT^2}\end{matrix}\right]\right\}$$

The critical temperature is determined as follows:

$$T_c' = \frac{T_0'}{\sqrt{1 - E'^2 / D'\lambda_{T_c'}'}} \tag{24}$$

For simplicity, let us assume $m_{h2} = X$, $m_{h--} = m_{Z2} = m_Q = K$. In order to have the first-order phase transition, the phase transition strength must be larger than 1, i.e., $\frac{\upsilon_{Xc}}{Tc'} \geq 1$.

If X is larger than 200 GeV, the heavy particle masses are in range of few TeVs in order to have the first-order phase transition (Phong *et al*., 2013). In order to have the first-order phase transition, if the contribution of h_2 with the mass is smaller than 200 GeV, K is smaller than 1.5 TeV (Phong *et al*., 2013).

The second/last step is the phase transition SU(2) → U(1). This phase transition does not involve the exotic quarks and boson $V^{\pm}$. Hence, in this case, v_χ is neglected and the contribution of $U^{\mp\mp}$ is equal to $W^{\mp}$. Then:

$$V^{eff}_{SU(2)\to U(1)} = \upsilon_0(\upsilon_\rho)\frac{1}{64\pi^2}\left(m^4_{h_2}\, In\frac{m^2_{h_2}}{Q^2} + 2m^4_{h++}\, In\frac{m^2_{h++}}{Q^2}\right)$$
$$+\frac{3}{64\pi^2}\left(\begin{array}{l} 2m^4_U\, In\frac{m^2_U}{Q^2} + m^4_{Z_1}\, In\frac{m^2_{Z_1}}{Q^2} + m^4_{Z_2}\, In\frac{m^2_{Z_2}}{Q^2} + 2m^4_W \\ In\frac{m^2_W}{Q^2} - 4m^4_t In\frac{m^2_t}{Q^2} \end{array}\right)$$
$$+\frac{T^4}{4\pi^2}\left[F-\left(\frac{m_{h2}}{T}\right) + 2F_-\left(\frac{m_{h++}}{T}\right)\right] + \frac{3T^4}{4\pi^2}$$
$$\left[2F-\left(\frac{m_U}{T}\right) + F_-\left(\frac{m_{Z_1}}{T}\right) + F-\left(\frac{m_{Z_2}}{T}\right) + 2F_-\left(\frac{m_W}{T}\right) + 4F_+\left(\frac{m_t}{T}\right)\right]$$

Denoting $V^{eff}_{SU(2)\to U(1)} \equiv V^{eff}_{SU(2)\to U(1)}(\upsilon_\rho, T)$, at high-temperature, it becomes:

$$V^{RM331}_{eff} = D(T^2 - T_0^2).\upsilon_\rho^3 - ET\left|\upsilon_\rho\right|^3 + \frac{\lambda_T}{4}\upsilon_\rho^4$$

where:

$$D = \frac{1}{24\upsilon_0^2}\left\{6m^2_W + 6m^2_U + 3m^2_{Z_1} + 3m^2_{Z_2} + 6m^2_t + m^2_{h_2} + 2m^2_{h\pm}\right\},$$
$$T_0^2 = \frac{1}{D}\left\{\begin{array}{l} \frac{1}{4}m^2_{h1} - \frac{1}{32\pi^2\upsilon_0^2}(6m^4_W + 6m^4_U \\ +3m^4_{Z_1} + 3m^4_{Z_2} - 12m^4_t + m^4_{h2} + 2m^4_{h\pm}) \end{array}\right\},$$
$$E = \frac{1}{12\pi v_0^3}(6m^3_W + 6m^3_U + 3m^3_{Z_1} + 3m^3_{Z_2} + m^3_{h_2} + 2m^3_{h\pm}),$$
$$\lambda_T = \frac{m^2_{h_1}}{2\upsilon_0^2}\left\{1 - \frac{1}{8\pi^2\upsilon_0^2 m^2_h}\left[\begin{array}{l} 6m^4_W\, In\frac{m^2_W}{bT^2} + 3m^4_{Z_1} \\ In\frac{m^2_{Z_1}}{bT^2} + 3m^4_{Z_2} In\frac{m^2_{Z_2}}{bT^2} + \\ 6m^4_U\, In\frac{m^2_U}{bT^2} - 12m^4_t In\frac{m^2_t}{b_F T^2} \\ +m^4_{h2} + 2m_{h\pm} In\frac{m^2_{h\pm}}{bT^2} \end{array}\right]\right\} \quad (25)$$

Here we have assumed $m_{H2} = m_{h--} = m_{Z2} \equiv Y$ with boson Z_2 and used $Q \equiv \upsilon_{\rho 0} = \upsilon_0 = 246$ GeV.

In order to have the first-order phase transition, the phase transition strength has to be larger than 1, i.e., $\frac{\upsilon_{\rho c}}{T_c} \geq 1$. The critical temperature T_c is given by:

$$T_c = \frac{T_0}{\sqrt{1 - E^2 / D\lambda_{Tc}}} \quad (26)$$

To survive the critical temperatures, T_c, T_0 must be positive, so T_0 is also positive, from which we can draw on conditions for heavy particles. Therefore, we get:

$$\frac{1}{4}m^2_{h1} - \frac{1}{32\pi^2\upsilon_0^2}(6m^4_W + 6m^4_U + 3m^4_{Z1} + 3m^4_{Z2} + m^4_{h2} + 2m^4_{h\pm}) > 0$$

With $m_{h1} = 125$ GeV and assuming $m_{Z2} = m_{h2} = m_{h--} = Y$, we can obtain Y < 344.718 GeV (Phong *et al*., 2013).

When $\frac{\upsilon_{\rho c}}{T_c} = 1$, i.e., $2E/\lambda_{Tc} = 1$, we obtain Y = 203.825 GeV and the critical temperature is in range $0 < T_c <$ 111.473 GeV. The contributions of new particles make of the strongly first-order phase transition that the SM cannot. However, there is one thing special, heavy particles as $U^{\pm\pm}$, h_2, h_{--}, Z_2 that contribute only the little part in their mass.

When temperature goes close to T_c, the second minimum slowly formed distinct, i.e., the phase transition nucleation appears.

When temperature goes over T_c, the minimum goes to zero, i.e., the symmetry phase is restored. This was showed that phase transition SU(2) → U(1) is the first-order phase transition (Phong *et al*., 2013).

We find that the effective potential of this model is different from that of the SM and it has contributions from heavy bosons as triggers for the strongly first-order phase transition with $m_{h1} = 125$ GeV.

We have got the following constraints on the mass of Higgs in RM331 (Phong *et al*., 2013):

$$285.56GeV < M_{h2} < 1.746TeV, 3.32TeV < M_{h--} < 5.61TeV$$

Thus we have used the effective potential at finite temperature to study the structure of the EWPT in the RM331 model. This phase transition is split into two phases, namely, the first transition is SU(3) → SU(2) or the symmetry breaking in the energy scale $\upsilon_{\chi 0}$ in order to generate masses for heavy particles and exotic quarks. The second phase transition is SU(2) → U(1) at $\upsilon_{\rho 0}$. The EWPT in this model may be the strongly first-order EWPT with $m_{h1} = 125$ GeV if the heavy bosons masses are some few TeVs.

Phase Transition in Economical 3-3-1 Model

In this section, we follow the same approach for E331 model (Ponce *et al.*, 2003), whose lepton sector is more complicated than that of the RM331 model. The E331 model has the right-handed neutrino in the leptonic content, the bileptons (two singly charged gauge bosons $W^{\pm}$, $Y^{\pm}$ and a neutral gauge bosons X^0), the heavy neutral boson Z_2 and the exotic quarks. The masses of particles in the E331 were summarized in Table 2.

As in the RM331, here EWPT takes place with two transitions: (i) SU(3) $\rightarrow$ SU(2) at the scale of ω_0 and the transition SU(2) $\rightarrow$ U(1) at the scale of υ_0 (Phong *et al.*, 2015).

The first phase transition SU(3) $\rightarrow$ SU(2) due to ω provides the bounds on parameters presented in Table 3.

The new bosons and exotic quarks can be triggers for the EWPT SU(3) $\rightarrow$ SU(2) to be the first-order. It was shown that the EWPT SU(2) $\rightarrow$ U(1) is the first-order phase transition, but it seems quite weak (Phong *et al.*, 2015).

Electroweak Sphalerons in the Reduced Minimal 3-3-1 Model

To be consistent with cosmological evolution, our strategy is the following: the model has to have an inflation or phase transition of the first-order. As a result, the leptogenesis or CP-violation exist. Then sphaleron completes to produce the BAU. Sphaleron is a transition at high temperature where thermal fluctuations can bring the magnitude of the Higgs field from zero VEV over the barrier to nonzero VEV classically without tunneling. In (Phong *et al.*, 2014), the sphalerons in the RM331 were considered. In the SM, the sphaleron rate is very small, about 10^{-60} (Klinkhamer and Manton, 1984; Akiba *et al.*, 1989; Moore, 1998; Farrar and Shaposhnikov, 1993 Arnold and McLerran, 1987); this rate is much smaller than the rate of BAU and smaller than the cosmological expansion rate.

To study the sphaleron processes, we consider the Lagrangian of the gauge-Higgs system:

$$L_{gauge--}Higgs = -\frac{1}{4}F_{\mu\nu}^{a\mu\nu} + (D_\mu X)^\dagger (D^\mu X) + (D_\mu \rho)^\dagger (D^\mu \rho) - V(X,\rho) \quad (27)$$

Assuming the least energy has the pure-gauge configurations ($F_{i,j}^a = 0$) functional in the temporal gauge:

$$\varepsilon = \int d^3X + \left[(D_\mu X)^\dagger (D^\mu X) + (D_\mu \rho)^\dagger (D^\mu \rho) + V(X,\rho)\right] \quad (28)$$

Table 2. Mass formulations of bosons in the E331 model

Bosons	$m^2(\omega, \upsilon)$	$m^2(\omega)$	$m^2(\upsilon)$
$m_W^2 \pm$	$\frac{g2}{4}\upsilon 2$	0	$80.39^2\ (GeV)^2$
$m_Y^2 \pm$	$\frac{g^2}{4}\left(\omega^2+\upsilon^2\right)$	$\frac{g^2}{4}\omega^2$	$80.39^2\ (GeV)^2$
$m_{X^0}^2$	$\frac{g^2}{4}\omega^2$	$\frac{g^2}{4}\omega^2$	0
$m_{Z_1}^2 \sim m_Z^2$	$\frac{g^2}{4c_w^2}\upsilon^2$	0	$91.68^2\ (GeV)^2$
$m_{Z_2}^2 \sim m_{Z'}^2$	$\frac{g^2c_w^2}{3-4s_w^2}\omega^2$	$\frac{g^2c_w^2}{3-4s_w^2}\omega^2$	0
$m_{H_1^0}^2$	$\left(2\lambda_2 - \frac{\lambda_3^2}{2\lambda_1}\right)\upsilon^2$	0	$125^2\ (GeV)^2$
$m_{H_1^0}^2$	$2\lambda_1 - \omega^2 + \frac{\lambda_3^2}{2\lambda_1}\upsilon^2$	$2\lambda_1\omega^2$	$\frac{\lambda_3^2}{2\lambda_1}\upsilon^2$
$m_{H_2^\pm}^2$	$\frac{\lambda_4}{2}\left(\omega^2+\upsilon^2\right)$	$\frac{\lambda_4}{2}\omega^2$	$\frac{\lambda_4}{2}\upsilon^2$

Table 3. The mass ranges of H_1^0 and $H_2^\pm$ for the first-order EWPT SU(3) $\rightarrow$ SU(2) and their upper bounds by the condition $m_{boson} < 2.2 \times T_c'$

ω [TeV]	T_c' [GeV]	$m_{H_1^\pm}$ [GeV]	$m_{H_2^\pm}$ [GeV]	Upper bound [GeV]
1	350	$0 < m_{H_1^\pm} < 300$	$0 < m_{H_2^\pm} < 720$	770
2	650	$0 < m_{H_1^\pm} < 600$	$0 < m_{H_2^\pm} < 1440$	1430
3	950	$0 < m_{H_1^\pm} < 900$	$0 < m_{H_2^\pm} < 2150$	2090
4	1300	$0 < m_{H_1^\pm} < 1200$	$0 < m_{H_2^\pm} < 2870$	2860
5	1600	$0 < m_{H_1^\pm} < 1500$	$0 < m_{H_2^\pm} < 3590$	3520

By the temperature expansion, the energy functional is given by:

$$\varepsilon = 4\pi\int_0^\infty d^3X\left[\frac{1}{2}(\nabla^2\upsilon_X)\frac{1}{2}(\nabla^2\upsilon_\rho)^2 + V_{eff}(\upsilon_X,\upsilon_\rho;T)\right] \quad (29)$$

In the static field approximation, we have two equations of motion for the VEVs in spherical coordinates (Phong *et al.*, 2014) for the VEVs:

$$\ddot{\upsilon}_X + \nabla^2\upsilon_X - \frac{\partial V_{eff}(\upsilon_X,T)}{\partial\upsilon_X} = 0 \quad (30)$$

And:

$$\ddot{\upsilon}_\rho + \nabla^2\upsilon_\rho - \frac{\partial V_{eff}(\upsilon_\rho,T)}{\partial\upsilon_\rho} = 0 \quad (31)$$

Then, the sphaleron energies in the SU(3) $\rightarrow$ SU(2) and SU(2) $\rightarrow$ U(1) phase transitions, are given, respectively:

$$\varepsilon_{sph.su(3)} = 4\pi\int\left[\frac{1}{2}\left(\frac{d\upsilon_X}{dr}\right)^2 + V_{eff}(\upsilon_X,T)\right]r^2dr \quad (31)$$

$$\varepsilon_{sph.su(2)} = 4\pi\int\left[\frac{1}{2}\left(\frac{d\upsilon_\rho}{dr}\right)^2 + V_{eff}(\upsilon_\rho,T)\right]r^2dr \quad (32)$$

The sphaleron rate per unit time per unit volume, Γ/V, is characterized by a Boltzmann factor, exp (-ε/T), as follows (Arnold and McLerran, 1987; 1988; Brihaye and Kunz, 1993):

$$\Gamma / V = a^4T^4exp(-\varepsilon / T) \quad (34)$$

where, V is the volume of the EWPT's region, T is the temperature, ε is the sphaleron energy and α = 1/30.

We will compare the sphaleron rate with the Hubble constant, which describes the cosmological expansion rate at the temperature T (Joyce, 1997; Onofrio *et al*., 2012):

$$H^2 = \frac{\pi^2gT^4}{90M_{pl}^2} \quad (35)$$

where, g = 106.75, $M_{pl} = 2.43\times10^{18}$ GeV.

Assuming that the VEVs of the Higgs fields do not change from point to point in the universe, then we have $\frac{d\upsilon_X}{dr} = \frac{d\upsilon_\rho}{dr} = 0$ and:

$$\frac{\partial V_{eff}(\upsilon_X)}{\partial\upsilon_X} = 0, \frac{\partial V_{eff}(\upsilon_\rho)}{\partial\upsilon_\rho} = 0 \quad (36)$$

Equation (36) shows that υ_χ and υ_ρ are the extremes of the effective potentials. The sphaleron energies can be rewritten as:

$$\varepsilon_{sph.su(3)} = 4\pi\int V_{eff}(\upsilon X,T)r^2dr = \frac{4\pi r^3}{3}V_{eff}(\upsilon_X,T)\Big|_{\upsilon_{Xm}} \quad (37)$$

And:

$$\varepsilon_{sph.su(2)} = 4\pi\int V_{eff}(\upsilon_\rho,T)r^2dr = \frac{4\pi r^3}{3}V_{eff}(\upsilon_\rho,T)\Big|_{\upsilon_{\rho m}} \quad (38)$$

where, $\upsilon_{\chi m}$, $\upsilon_{\rho m}$ are the VEVs at the maximum of the effective potentials. From (37) and (38), it follows that the sphaleron energies are equal to the maximum heights of the potential barriers.

The universe's volume at a temperature T is given by $V = \frac{4\pi r^3}{3} = \frac{1}{T^3}$. Because the whole universe is an identically thermal bath, the sphaleron energies are approximately:

$$\varepsilon_{sph.su(3)} \sim \frac{E'^4T}{4\lambda'^3_T};\varepsilon_{sph.su(2)} \sim \frac{E'^4T}{4\lambda'^3_T} \quad (39)$$

From the definitions (37) and (38), the sphaleron rates take the form, respectively:

$$\Gamma_{su(3)} = \alpha_w^4T\,exp\left(-\frac{E'^4T}{4\lambda'^3_TT}\right) \quad (40)$$

And:

$$\Gamma_{su(2)} = \alpha_w^4T\,exp\left(-\frac{E'^4T}{4\lambda'^3_TT}\right) \quad (41)$$

For the heavy particles, E, λ, E′ and λ′ are constant and the sphaleron rates (for the the phase transition SU(2) → U(1)) in this approximation are the linear functions of temperature (Phong *et al*., 2014)

Thus, the upper bounds of the sphaleron rates are much larger the Hubble constant (Phong *et al*., 2014):

$$\Gamma_{su(3)} \sim 10^{-3} \gg H;\Gamma_{su(2)} \sim 10^{-4} \gg H \sim 10^{-13} \quad (42)$$

In a thin-wall approximation, sphaleron rates are presented in Tables 4 and 5.

Table 4. The sphaleron rate in the EWPT SU(3) → SU(2) with $m_q(v_\chi) = m_{h2}(v_\chi)$ = 1500GeV

T [GeV]	$R_{b.su(3)}$ [$10^{-6}\times GeV^{-1}$]	$R_{b.su(3)}/\Delta l'$	$\varepsilon_{sph.SU(3)}$ [GeV]	$\Gamma_{SU(3)}$ [$10^{-11}\times GeV$]	H [$10^{-12}\times GeV$]	$\Gamma_{SU(3)}$/H
1479.48(Γ_1')	10.0	10.0	6975.1700	1.63719×10^6	3.08195	5.31×10^6
1450	12.0	12.0	12481.300	3.2702×10^4	2.96034	1.10×10^5
1400	13.0	13.0	17206.300	7.94481×10^2	2.75970	2.878×10^3
1390	15.0	15.0	23251.700	9.3264	2.72042	3.42
1388.4556 (Γ_c')	16.5	16.5	28135.100	0.2714	2.71438	1.00
1387	17.0	17.0	29854.000	0.07687	2.70869	0.28
1000	19.0	19.0	60590.800	5.98×10^{-19}	1.40801	4.25×10^{-18}
900	22.0	22.0	89250.800	9.50×10^{-36}	1.14049	8.33×10^{-35}
865.024 (Γ_0')	25.0	25.0	119110.36	1.69×10^{-52}	1.05357	1.60×10^{-51}

Table 5. The sphaleron rate in the EWPT SU(2) → U(1) with $m_{h2}(\upsilon_\rho) = 100GeV$, $m_{h\pm\pm}(\upsilon_\rho) = 350GeV$

T $[GeV]$	$R_{s.su(2)}$ $[10^{-4} \times GeV^{-1}]$	$R_{s.su(2)}/\Delta l$	$\varepsilon_{sph.SU(2)}$ $[GeV]$	$\Gamma_{SU(2)}$ $[10^{-12} \times GeV]$	H $[10^{-14} \times GeV]$	$\Gamma_{SU(2)}/H$
141.574 (T_1)	6.0	10	742.838	919936.07000000	2.82211	3.25×10^{7}
141.5	8.0	10	1020.870	128525.28000000	2.81916	4.55×10^{6}
141	10.0	10	1442.750	6264.89000000	2.79927	2.23×10^{5}
140	12.0	12	2342.210	9.37289000	2.75970	339.6000000
138.562 (T_c)	13.1	13	3135.750	0.02703000	2.70300	1.0000000
137	14.0	14	3922.290	0.00006220	2.64270	2.357×10^{-3}
130	16.0	16	6567.080	1.847×10^{-14}	2.37900	7.76×10^{-13}
120	18.0	18	10068.200	5.403×10^{-29}	2.02754	2.66×10^{-27}
118.42 (T_0)	20.0	20	12656.700	5.595×10^{-39}	6.20900	9.01×10^{-38}

Here $R_{b.su(3)}$ and $\Delta l'$ are respectively the radius and the wall thickness of a bubble which is nucleated in the phase transitions.

We conclude that the sphaleron rates are larger than the cosmological expansion rate at temperatures above the critical temperature and are smaller than the cosmological expansion rate at temperatures below the critical temperature. For each transition, baryon violation rapidly takes place in the symmetric phase regions but it also quickly shuts off in the broken phase regions. This may provide B-violation necessary for baryogenesis, as required by the first of Sakharov's conditions, in the connection with non-equilibrium physics.

Conclusion

In this review, we have showed that the 3-3-1 models are able to describe the cosmological evolution. The 3-3-1 models contain the hybrid inflationary scenario and the first-order phase transitions. The inflation happens in the GUT scale, while phase transition has two sequences corresponding two steps of symmetry breaking in the models. The sphaleron rates are much larger than the Hubble constant. They are larger than the cosmological expansion rate at temperatures above the critical temperature and are smaller than the cosmological expansion rate at temperatures below the critical temperature. From these considerations, some bound on model parameters are deduced.

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Ethics

This is an original work, containing unpublished material and supplied as arXiv:1501.01852. The author has read and approved its publication with no ethical issues involved.

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